An Econometric and Neural Network Approach to the Impact of Income Distribution on the Economy

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Abstract

Since 1967 the Gini coefficient as a measure of income distribution reported by the United States government has had an upward trend, suggesting a more concentrated income distribution in the economy. The purpose of this study is to investigate the impact of income distribution on the economy by using a reduced-form econometric model similar to the St. Louis equation, and a neural network method.

The results of both methodologies with different mixes suggest that a more concentrated income distribution has an adverse effect on the GDP in the short run, but a positive impact in the long run. Also, the simulation of the econometric model and the neural network showed lower forecast errors for the neural network than those of the St. Louis equations.

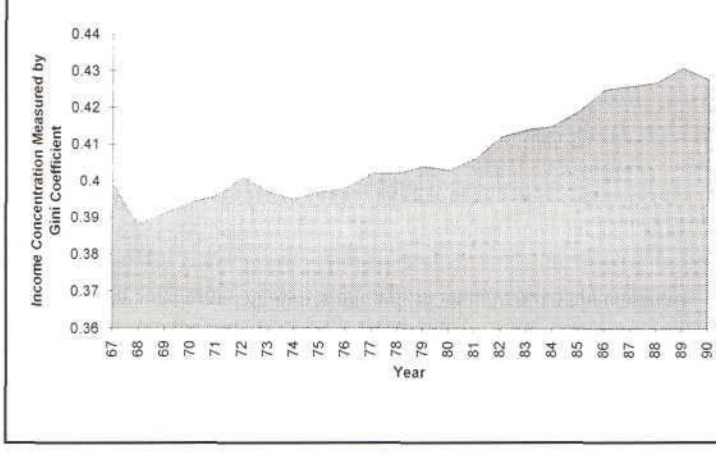
Introduction

In recent years the United States economy has experienced slower economic growth, which is a major concern to economists, politicians, and others. Several factors have contributed to this sluggish growth. The foremost cited by economists are: slower growth of the labor force due to the recent baby bust, a lower saving rate and less capital formation, and lower labor productivity growth. More recently, the end of the cold war, with the resulting defense cuts, and the inability of the federal government to stimulate the economy due to the high deficit and national debt, have added to growth problems.

Another possible factor contributing to economic stagnation is recent changes in the distribution of income, or income inequality. According to the most recent Census Bureau report, between 1967 to 1974 both the number of Americans living in poverty and the rate of poverty declined. However, between 1974 to 1992 both of these measures rose. In 1992 the number of poor people rose for the third year in row, to 36.9 million, or 14.5 percent of the population.[1] Poverty rates persisted slightly above 20 percent for people under 18, and slightly above 10 percent for those above 64. In 1992, the latest year for which data are available, the average income of the top 5 percent of American (those who make an average of $145,000 a year) rose by $3,500. In that year, "the rich got richer," but the poor and the middle class stayed the same.

Economists use the Gini coefficient as a measure of income inequality.[2] Figure 1 shows the Gini index for 1967 to 1990. The index increased from 0.399 in 1967 to 0.402 in 1977. Thereafter, the index increased to 0.414 in 1983, and by 1990 it was 0.428.[3]

Figure 1.
Gini Coefficient as a Measure of Income Inequality: 1967 - 1990



The link between income distribution and business cycles and economic activity have been investigated by different economists. The purpose of this study is to investigate the effect of higher income inequality on the economy, as measured by Gross Domestic Product (*GDP*). Two distinct methodologies are used for this purpose. First, a simple reduced-form econometric model, very much similar to the St. Louis equations, and second, a new method called the neural network, are employed. The results of all versions of both the econometric and neural network methods suggest that a higher income inequality has an adverse effect on *GDP* in the short run. However, it has a positive effect on *GDP* in the long run.

After the methodology of the econometric method and the neural network are described their empirical results are explained and compared. Next, findings of the simulation of the models and the errors of their forecasts are explained and compared. Finally a summary and conclusions are presented.

Models of the Investigation

Econometric Models

The structural-form econometric models of the United States economy which are used for forecasts of the *GDP*, and other macro variables are complicated, with different sectors and a large number of equations. Their use is beyond the scope of this paper. The St. Louis models are in the reduced form and much simpler; thus, their use for this study is appropriate.[4] In this study, two revised versions of the St. Louis models are used, and their estimated results are compared with those of the neural network.

First one is a first-difference specification in the form of:

$$\Delta Y_t = a + \sum_{i=0}^{m} b_i \, \Delta GC_{t-i} + \sum_{i=0}^{n} c_i \, \Delta G_{t-i} + \sum_{i=0}^{k} d_i \, \Delta M_{t-i} + e_t, \tag{1}$$

where

- Y = Nominal *GDP*
- M = Money supply, M1
- G = High-employment federal government expenditures
- GC = Gini coefficient
- e = error term,

and D is the first difference; for instance, $\Delta Y = Y_t - Y_{t-1}$, and $\Delta G = G_t - G_{t-1}$. The coefficients a, b, c, and d are the parameters of the model, and m, n, and k are the lag structures of the variables in the model for the short run and the long run. According to equation (1), the current changes in annual output, ΔY, depend on the current annual changes in the Gini coefficient (GC), the money supply (M), and annual changes in high-employment federal government expenditures (G), the last two annual changes of the Gini coefficient, and the last annual change of the other two policy variables. The model assumes that the impact of the money supply and government expenditures is constrained, with both endpoint parameters to equal zero.

The second version of the model is in the form of growth specification, as

$$\Gamma Y_t = a + \sum_{i=0}^{m} b_i \, \Gamma GC_{t-i} + \sum_{i=0}^{n} c_i \, \Gamma M_{t-i} + \sum_{i=0}^{k} d_i \, \Gamma G_{t-i} + e_t \tag{2}$$

Here Γ represents the rate of growth of variables. According to this equation, the growth of *GDP* depends on the current and two annual lags of the growth of Gini coefficient, and current and the one-year lag of the growth of money supply and federal government expenditures. Again, the model assumes that the impact of money supply and government expenditures on *GDP* follows an Almon distributed lag polynomial, and that the impact is constrained, with both endpoint parameters to equal zero.

In sum, the Gini coefficient, as another independent variable, is added to two versions of the St. Louis models in the form of equations (1) and (2), representing the econometric models of this research. Their findings are used for comparison with those of the neural network.

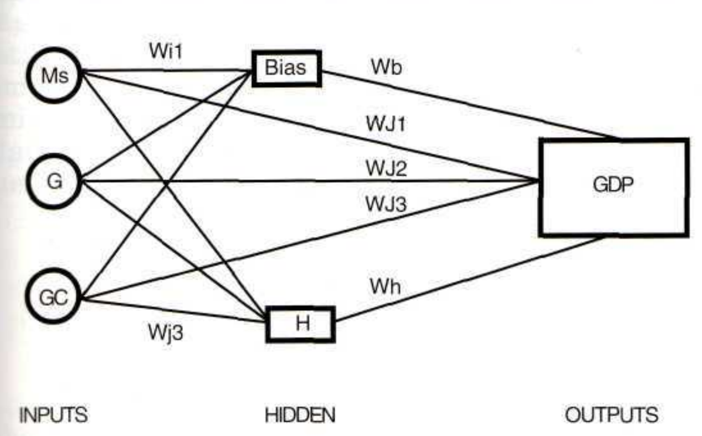
Neural Networks

The study of neural networks is a branch of the field of artificial intelligence. Like the structure of the brain, neural networks are multiple-layer configurations consisting of simple processing elements (*PE*'s) that interact with each other through weighted connection in the system. A typical neural network consists of: 1) an input layer, 2) one or more intermediate or hidden layers, and 3) one output layer.

A general design of an artificial neural network, with three inputs, a bias, a hidden layer and one output, the design used in this study, is shown in Figure 2. In this neural network set, income inequality is regarded as another input, in addition to the traditional monetary and fiscal policy variables affecting the *GDP* as output. It should be noted that the inputs and outputs of the neural network are similar to those of the econometric models.

As shown in the figure, each connection has a weight, and each element computes a weighted sum of the incoming values and passes this sum through a nonlinear or linear function as output. The hidden layer(s) is a processor and a bridge between the inputs

Figure 2.
Structure of a Neural Network with Three Inputs, One Hidden Layer, One Bias, and One Output



and output(s). The mathematical function of the hidden layers is in different forms. In this inquiry, a neural network consisting of different mixtures is used. However, in all versions, *GDP* was used as output and the Gini coefficient (*GC*), M1 money supply (*M*), and federal government expenditure (*G*) were set as inputs.

Inputs of this network are connected both directly and indirectly (through the hidden layer) to the output (Figure 2). Furthermore, there is a bias processing element (*PE*) that is connected to the output. The bias *PE* is similar to the intercept concept in equations of econometric models. It absorbs noises of other contributors to the output in the system. Neural networks, unlike econometric models, which have to pass some degree of assumptions and therefore become parametric, are "nonparametric" in nature, even though the weights of connections are parameterized.

The three main phases in the operation of a network are Learning, Recall and Testing. In the learning phase, the neural network recognizes a pattern between inputs and output and estimates final output, called "actual." Subsequently, this output is compared with the desired output, and an error is calculated. The error becomes a factor to adjust the weights and, subsequently, a factor to reduce the error and readjust the weights of the connections. This process continues further until the errors decline to an acceptable level, if possible. Through this process, the neural network learns the rules and adapting patterns for processing the knowledge. The resulting pattern of linking the inputs to the output in the learning process can be used for testing the accuracy of the networks. In addition, the pattern can be used for prediction of the output, given the inputs.

The most widely used neural network model is back propagation, which is an algorithm for adjusting connection weights in a multiple layer network. Back propagation is a learning design by which the multilayer network is set for pattern recognition, utilizing actual cross section or time-series data as the external teacher. The "explain" command reveals the information about the causes and the magnitudes of contributions of inputs on the output. This information is somewhat similar to the measure of elasticity of output for each input in the econometric approach.[5]

In summary, for both the econometric models and the neural networks setup, *GDP* is the output (independent variable), and the Gini coefficient, money supply, and government spending are the inputs (independent variables).

Empirical Results

Econometric Method: Short Run

Data for the Gini coefficient were only available in the forms of annual figures from 1967 to 1990. Two mixtures of inputs and output were used to estimate equations (1) and (2). Their findings are shown in Tables 1 and 2. Because annual data is used, only two types of lag structures were used for the estimation of the model in the short run. In the first specification, only the current variables without a lag, as in equations (1) $m = n = k = 0$, were used. In the second specification current data of the Gini coefficient, and a one year lag of monetary and fiscal policy data, $m = 0$, and $n = k = 0, 1$, were used.

Table 1 shows the estimated results of equation (1) for the short-run impact. The resulting signs of the Gini coefficient (*GC*) for both mixes are negative, suggesting that a higher income concentration has an adverse effect on *GDP* in the short run. That is, more concentration of income would cause a slow down in *GDP* in the short run. The estimated coefficients of the Gini coefficient is higher for the version with a lag than for the version without a lag of fiscal and monetary variables.

Table 2 presents the estimated findings of equation (2) for the shot-run effect. Again, as the resulted negative signs of the Gini coefficient indicate a higher growth of income concentration has an adverse effect on the growth of *GDP* in the short run. Here, the estimated coefficient of the Gini coefficients is higher for the version without a lag than the one with a lag of *GC*. This suggests that the immediate effect of income inequality is stronger, but its effect declines over time.

Thus, the findings of the econometric models imply that in the short run more income-inequality would slow down the economy. As expected, the signs of fiscal and monetary policy variables for all four mixes are positive.[6]

Table 1:
Results of the Impact of Income Distribution on GDP:
First-Difference Version of Econometric Model

Variables	Coef	*t*-Statistics	Variables	Coef	*t*-Statistics
Constant *	111.3	1.9968	Constant	100.49	1.8609
ΔGC	**- 3.20**	**-0.4681**	**ΔGC**	**- 5.528**	**-0.839**
ΔM_0	1.2321	1.2763	ΔM_0	0.9837	1.9634
			ΔM_1	0.9837	1.9634
			Sum ΔM	1.9674	1.9633
ΔG_0	1.5997	1.9376	ΔG_0	0.7597	1.6509
			ΔG_1	0.7597	1.6509
			Sum ΔG	1.5194	1.6509
R^2	0.2689		R^2	0.343592	
Adj R^2	0.1227		Adj R^2	0.21231	
DW	1.6971		*DW*	1.6965	
SE	79.364		*SE*	75.204	
F(3,15)	1.8395		*F*(3,15)	2.61721	
No of Obs	19		No of Obs	19	

* Δ is first differences,

GDP is output (dependent variable), current Gini coefficient (*GC*), current M1 money supply (M_0), one-year lag M1 money supply (M_1), current- high employment government expenditures (G_0), and one-year lag high employment government expenditures (G_1) are inputs (independent variables).

Table 2:
Results of the Impact of Income Distribution on GDP:
Growth Without Lag Version of Econometric Model

Variables	Coef	*t*-Statistics	Variables	Coef	*t*-Statistics
Constant *	4.7146	1.5179	Constant	5.1335	1.4551
ΓGC_0	**-0.30**	**-1.35**	**ΓGC_1**	**-0.07**	**-0.26**
ΓM_0	0.0985	0.7276	ΓM_0	0.0873	0.5624
ΓM_1	0.0985	0.7276	ΓM_1	0.0873	0.5624
Sum ΓM	0.1969	0.7276	Sum ΓM	0.17467	0.5624
ΓG_0	0.1624	1.626	ΓG_0	0.1322	1.1234
ΓG_1	0.1624	1.626	ΓG_1	0.1322	1.1234
Sum ΓG	0.32487	1.6259	Sum ΓG	0.26431	1.1234
R^2	0.227446		R^2	0.103657	
Adj R^2	0.061899		Adj R^2	-0.08842	
DW	1.8663		*DW*	1.904	
SE	2.54926		*SE*	2.6963	
F(3,14)	1.17763		*F*(3,14)	0.4626	
No of Obs	18		No of Obs	18	

* Γ is the rate of growth,

GDP is output (dependent variable), current Gini coefficient (GC_0), one-year lag Gini coefficient (GC_1), current M1 money supply (M_0), one-year lag M1 money supply (M_1), current high employment government expenditures (G_0), one-year lag high employment government expenditures (G_1) are inputs (independent variables).

Neural Network Method: Short Run

The back propagation technique, a Delta-learning rule, and a Sigmoid transfer function were considered appropriate for the learning and pattern recognition. Again, only annual data from 1967 to 1990 were utilized. The combinations of data for the short-run impact of income concentration on *GDP* give two mixtures of (1) first difference (Δ), and rate of growth data (Γ). In both mixes, inputs and output of the network were as follows:

Inputs: Current data of Gini coefficient (*GC*), M1 money supply (*M*), and high-employment government expenditures (*G*).

Output: Current data of Gross Domestic Product (*GDP*).

Table 3 presents the results of the learning of the neural network for each of these two mixtures. The input value column represents results of the current value of the *PE* where the connection originates. The estimated weights of the connection between the inputs (*PE*) and the output (*GDP*) are also shown in the table.[7]

Table 3: Result of the Impact of Income Distribution on the GDP Resulted Input Value, Weight, and Delta Weight of the Neural Network

Mixture	Input PC	Input Value	Weight	Delta Weight
Δ*	Bias	+1.0000	-0.7188	+0.0000
	Δ*GC*	**+0.7647**	**-0.1017**	**+0.0000**
	Δ*M*	+5919	+0.7360	+0.0000
	Δ*G*	+0.5800	+1.4542	+0.0000
	Hidden	+0.4164	-0.3826	+0.0000
Γ	Bias	+1.0000	-0.0579	+0.0057
	Γ*GC*	**+1.0000**	**-0.5944**	**+0.0057**
	Γ*M*	+0.4490	+0.4381	-0.0029
	Γ*G*	+0.4383	+1.8595	+0.0023
	Hidden	+0.4258	-0.0374	+0.0024

*Δ= First differences, Γ= The rate of growth, *GDP* is output, current Gini coefficient (*GC*), M1 money supply (*M*), high employment government expenditures, are inputs.

The estimated signs of the weight of the current Gini coefficient for both the first-difference and the rate of growth mixtures are negative. These findings, similar to and consistent with those of the econometric models, suggest that a more concentrated income distribution has adverse effect on *GDP* in the short run. The weight of the Gini coefficient for the first-difference mix is lower than those of the fiscal and monetary variables. However, the weight of the Gini coefficient for the growth mix is higher than that of the first-difference.

As expected, the resultant signs of the weights of the money supply (*M*) and the high-employment government expenditures (*G*) are positive for both the first-difference and the growth mixtures. The input values of the bias are higher than those of other *PE*'s for both versions, which implies that the influence of other non-policy variables, the so called "Z-factors," are relatively greater than the three variables in the model.

The neural network also calculated the relative sensitivity of *GDP* for the Gini coefficient and each policy variable. Table 4 shows the resultant average percentage changes of *GDP* as a consequence of a five percent change in each of the three variables. This concept is the nonlinear version of the elasticity measure.[8] Correspondingly, each 5 percent increase in the *GC* would reduce the level of *GDP* by 2 percent (according to the first-difference model estimate), and reduce the rate of growth of *GDP* by over 14 percent (according to the growth model estimate).

Table 4 also presents variations of the elasticity measures of those exogenous variables, measured by their average standard deviations. The low standard deviations of all the inputs suggest that the estimated coefficients for these variables are very concentrated and do not have large variations. Indeed, some of these standard deviation numbers are very close to zero.

Table 4: The Results of the Neural Networks' Average, and Standard Deviation of GDP Elasticity of Gini Coefficient, Government Expenditures, and Money Supply, with Dithing of 5%

Mixes	Measures	Gini Coefficient	Government Expenditures	Money Supply
Δ*	Average	-2.0191	32.7669	16.5982
	Standard Dev	0.00067	0.09114	0.04645
Γ	Average	-14.281	20.7083	10.5666
	Standard Dev	0.76808	1.11346	0.56837

* Δ = First differences, Γ = The rate of growth

Econometric Method: Long Run

To investigate the long-run effect of income distribution on *GDP*, the order of the lags of the Gini Coefficient was increased for both the first-difference and the growth model. Again using annual data from 1967 to 1990, equations (1) and (2) with $m = 0, 1, 2$, and $n = k = 0, 1$, were estimated. Their results are shown in Table 5. The estimated signs of the coefficients of the current Gini coefficient (GC_0) for both models are negative. This outcome is consistent with the short-run results reported earlier. However, the estimated signs of the one year lag (GC_1) and two-year lag (GC_2) of the Gini coefficient for both versions are positive, suggesting the positive impact of income concentration on the economy in the long run.

Neural Network Method: Long Run

The neural network set with the same annual data from 1967 to 1990 with current and up to two year lags of the *GC* was used. Their estimated results are shown in Table 6. The signs of the weights of the current Gini coefficient (GC_0) for both the first-difference and the growth models are negative. Again these findings are consistent with the short-run results of the earlier findings. However, the signs of the weights of the two-year lag of the Gini coefficient (GC_2) for both versions and the one-year lag of the first-difference version are positive, suggesting the positive consequence of income concentration on the economy in the long run.

To summarize, the findings of both the econometric methods and neural networks are consistent. They imply that a higher income concentration would slow down the *GDP* in the short run, and would cause its expansion in the long-run.[9]

Simulation and Comparison

To test the accuracy and validity of neural network forecasts, compared to those of the econometric method, total data of 23 observations (1967 to 1990) were divided into two parts. The first part, consisting of 13 observations (1967 to 1980), was used for the estimation of the parameters of both the econometric models and the learning of the

Table 5: Results of the Impact of Current and Lag Income Distribution on GDP: First Difference and Growth Versions of Econometric Model

Variables	Coef	*t*-Statistics	Variables	Coef	*t*-Statistics
Constant *	47.26	0.97	Constant	4.89	1.42
ΔGC	**-769.2**	0.12	ΓGC	**-0.58**	0.53
ΔGC_1	**4645.1**	0.59	ΓGC_1	**0.39**	0.24
ΔGC_2	**16604.0**	2.36	ΓGC_2	**1.15**	0.88
ΔM_0	0.235	0.21	ΓM_0	0.043	0.15
ΔM_1	0.235	0.21	ΓM_1	0.043	0.15
Sum ΔM	0.470	0.21	Sum ΓM	0.086	0.15
ΔG_0	2.397	3.22	ΓG_0	0.330	1.25
ΔG_1	2.397	3.22	ΓG_1	0.330	1.25
Sum ΔG	4.794	3.22	Sum ΓG	0.660	1.25
R^2		0.544	R^2		0.202
Adj R^2		0.382	Adj R^2		0.083
DW		1.626	*DW*		1.642
SE		69.78	*SE*		2.68
$F(3,14)$		3.3	$F(3,14)$		0.71
No of Obs		20	No of Obs		20

*Δ is first differences, Γ is the rate of growth.

GDP is output (dependent variable), current Gini coefficient (GC_0), one-year lag Gini coefficient (GC_1), two-year lag Gini coefficient (GC_2), current M1 money supply (M_0), one-year lag current M1 money supply (M_1), two-year lag current M1 money supply (M_2), current high employment government expenditures (G_0), one-year lag high employment government expenditures (G_1), two-year high employment government expenditures (G_2) are inputs (independent variables).

Table 6:
Neural Network Results of the Short-Run and Long-Run Impact of Income Distribution on GDP

Mixture	Input PE	Input Value	Weight	Delta Weight
Δ*	Bias	+1.0000	-01.4511	-0.0017
	ΔGC	**+0.3000**	**-0.0811**	**-0.0005**
	ΔGC_1	**+0.6000**	**+0.1770**	**-0.0010**
	ΔGC_2	**+0.6471**	**+1.4902**	**-0.0012**
	ΔM	+0.2173	+0.2931	-0.0004
	ΔG	+1.0000	+1.2388	-0.0020
	Hidden	+0.4154	-0.8466	+0.0007
Γ	Bias	+1.0000	-0.0524	+0.0045
	ΓGC	**+0.8105**	**-0.5586**	**+0.0033**
	ΓGC_1	**+0.5040**	**-0.0853**	**+0.0024**
	ΓGC_2	**+0.7712**	**+0.8869**	**-0.0033**
	ΓM	+0.5335	+0.3239	+0.0023
	ΓG	+0.4504	+0.9463	+0.0020
	Hidden	+0.4055	-0.3238	+0.0019

*Δ= First differences, Γ= The rate of growth,
GDP is output, current Gini coefficient (*GC*), one-year lag Gini coefficient (GC_1), two-year lag Gini coefficient (GC_2), M1 money supply (*M*), high employment government expenditures, are inputs.

neural network. The second part, consisting of 10 observations (1981 to 1990), was used to simulate and compare the performance of these two methods. Using the parameters of the short-run econometric equations and the inputs of the second part data, the output of the second series was estimated and compared with the desired data. Similarly, the parameters of the neural network's learning run and the input of the second part were used to forecast the output of the second part and compared with the desired data. Then, the errors of the forecasts by both methods were calculated.

The mean-square error (*MSE*) and standard deviations of the forecast of the econometric models and the neural network were calculated and are shown in Table 7. The mean-square errors of the forecast from the neural network approach for both the first-difference and growth versions are smaller than those of the econometric method. The results of the simulation of the models suggest that the neural network forecast is more accurate, with a smaller error, than those of the St. Louis version of the econometric approach.

Table 7:
Mean Square and Standard Deviation of the Forecast Error by Neural Network and Econometric Models

Mixtures	Neural Network Models	Econometric Models
First-Diff		
MSE*	2920.254	3891.918
St. Dev.	57.47364	81.74642
Growth		
MSE	8.4808	8.65473
St. Dev.	1.4103	1.238648

* MSE = Mean Square Error, St. Dev. = Standard Deviation.

Summary and Conclusions

The purpose of this study was to investigate the role and the influence of income distribution on *GDP*. The St. Louis version of the econometric models, and the neural network method, a branch of artificial intelligence, were used for this purpose. The results of both methodologies for the first-difference and the growth versions of the data are consistent and suggest the same conclusions: that a higher income concentration has an adverse effect on *GDP* in the short run, but an expansionary effect in the long run. Thus, increased income concentration can slow down the economy and may cause higher unemployment in the short run, but it can cause faster growth of the economy in the longer run.

The findings of the simulation of both methods indicate that the forecasting *GDP* by the neural network method is more accurate than the use of econometric models. It is recommended that economists and forecasters should use the neural network as a strong tool and compare their results with those of the econometric methods

Endnotes

Special thanks to Profesor Jean Caldwell for reading the final draft of this paper and making editorial changes. Also, I am grateful to the editor of *The Central Business Review* and an anonymous referee for their constructive suggestions. The usual disclaimer applies.

[1] The poverty threshold for one person in 1992 was $7,143, and for a family of four was $14,335.

[2] The Gini coefficient is a ratio of the area between the 45-degree line and the Lorenz curve to the area between the 45-degree line and the axes. That is, it is the ratio of the deviation of actual distribution from equal distribution of income to equal distribution of income.

[3] The Gini coefficient (*GC*) was regressed on time (Year, *YR*) for 1967 to 1990 (24 observations) the results of the equation, with *t* ratios in parentheses, is:

$$GC = -3.0217 \quad + 0.00173\, YR$$

$$(11.609) \qquad (13.173)$$

$$R^2 = 0.888 \quad \text{Adj } R^2 = 0.882$$

The high magnitude of the *t*-ratio of *YR* suggests that the upward trends of the Gini coefficient is statistically significant.

[4] The first version of the model was introduced in 1968 by Andersen and Jordan in the form of first difference as follows:

$$\Delta Y_t = a + \sum_{i=0}^{4} b_i \Delta M_{t-i} + \sum_{i=0}^{4} c_i \Delta G_{t-i} + \sum_{i=0}^{4} d_i \Delta R_{t-i} + e_t . \tag{N1}$$

Equation (N1) as a reduced-form model assumes that the current changes in quarterly output, ΔY, depend on the current quarterly changes in the M1 money supply (M_1), current quarterly changes in high-employment federal government expenditures (G), high-employment federal government revenues (R), and the last four quarterly changes of these three policy variables. Furthermore, the model assumes that the impact of these exogenous variables follows an Almon (1965) distributed lag of fourth degree polynomial, and that the impact is constrained with both endpoint parameters to equal zero.

In the later version of the model, the high-employment federal government revenue in equation (N1), term R, was dropped (Anderson and Carlson, 1970), and presented as

$$\Delta Y_t = a + \sum_{i=0}^{4} b_i \Delta M_{t-i} + \sum_{i=0}^{4} c_i \Delta G_{t-i} + e_t . \tag{N2}$$

Finally, the model presented in the form of growth specification (Carlson, 1980) as

$$\Gamma Y_t = a + \sum_{i=0}^{4} b_i \Gamma M_{t-i} + \sum_{i=0}^{4} c_i \Gamma G_{t-i} + e_t , \tag{N3}$$

where Γ represents the rate of growth of variables. According to this equation, the growth of *GDP* (*Y*) depends on the current and four quarter lags of the growth of the money supply (*M*) and the current and four quarter lags of the growth of federal government expenditures (*G*). Again, the model assumes that the impact of these two variables follows an Almon type distributed lag of fourth degree polynomial, and that the impact is constrained with both endpoint parameters to equal to zero.

Immediately after the publication of the Andersen-Jordan model in 1968, the model was attacked on several grounds. First, it was argued that reduced form models are not valid econometrically. Second, the model was criticized on the ground of misspecification, in that important variables (such as investment, consumption, and exports) are omitted. Third, critics argued that the policy variables are not strictly exogenous. Finally, critics attacked the model's conclusions that monetary policy is more powerful than fiscal policy, and that the latter has no lasting effect on nominal GNP. Since that publication, both sides of the debate have grown to believe that the role and the importance of other non-policy variables are even more important than those of the policy variables. These were called "Z-factors," and their relative magnitude has been estimated and recognized before.

[5] The network calculates the summation of multiplication of inputs by their corresponding weights as follows:

$$X_j^{[s]} = f_i\left(\Sigma\left(X_{ji}^{[s]} * W_{ji}^{[s-1]}\right)\right) = f\left(I_j^{[s]}\right). \tag{N4}$$

Where $X_j^{[s]}$ is the current output state of *j*th neuron in layer *s*, $W_{ji}^{[s]}$ is the weight on connection joining *i*th neuron in layer [*s*-1] to *j*th neuron in layer *s*, and $I_j^{[s]}$ is the weighted summation of inputs to *j*th neuron in layer *s*.

This weighted summation is then transformed by a transfer function to the hidden layer. Three common non-linear transfer functions used in artificial neural networks are the Sigmoid, hyperbolic tangent, and sine functions. The Sigmoid transfer function, which is used in this study, is a continuous monotonic mapping of the input into a value between zero and one. The Sigmoid function for the local error, e, is a smooth version of {0,1} step function, and is defined as

$$f(z) = (1 + e^{-z})^{-1} . \tag{N5}$$

The local error at processing element j in level s is determined by

$$e_j^{[s]} = \delta E / \delta(I_j^{[s]}). \tag{N6}$$

Here E is the global error function which is differentiable of all the connection weights in the network. Using the chain rule twice in succession gives a relationship between the local errors at a particular processing element at level s and at the level $s + 1$ as

(N7)

$$e_j^{[s]} = f'(I_j^{[s]}) * \Sigma (e_k^{[s+1]} * W_{kj}^{[s+1]}).$$

Since $f'(z) = f(z) * (1.0 - f(z))$ from equation (N4), equation (N7) can be rewritten as

(N8)

$$e_j^{[s]} = x_j^{[s]} * (1.0 - x_j^{[s]}) * \Sigma (e_k^{[s+1]} * W_{kj}^{[s+1]}).$$

The summation term in (N8) which is used to back-propagate errors is analogous to the one in (N4) which is used to forward propagate the input through the network.

To increment or decrement the weights in order to decrease the global error, a gradient descent rule is used as follows:

(N9)

$$DW_{ji}^{[s]} = - lcoef * (\delta E / \delta[w_{ji}^{[s]}]).$$

where *lcoef* is a learning coefficient, and the partial derivative in (N9) is

(N10)

$$\delta E / \delta w_{ji}^{[s]} = \delta E / \delta I_j^{[s]} * (\delta I_j^{[s]} / \delta[w_{ji}^{[s]}])$$
$$= - e_j^{[s]} * x_i^{[s-1]}$$

and combining (N9) and (N10) together gives

(N11)

$$DW_{ji}^{[s]} = lcoef * e_j^{[s]} * x_i^{[s-1]}.$$

Suppose the desired output, D, is specified by a teacher, and the actual output, A, is produced by the network with its current set of weights. Then the global error function, E, is the square of the Euclidean distance between the desired output and the actual output of the network for a particular input pattern, and is given by

(N12)

$$E = 0.5 * \Sigma (D_k - A_k)^2$$

where $(D_k - A_k)$ is the raw local error. From (N6), the scaled "local error" at each processing element of the output layer is determined by

(N13)

$$e_k^{(0)} = - \delta E / \delta I_k^{(0)} = - (\delta E / \delta A_k) * (\delta A_k / \delta I_k)$$
$$= (D_k - A_k) * f'(I_k).$$

The sum of all pattern specific error function is the overall global error function. The back-propagation algorithm modifies the weights to reduce the particular component of the overall error function.

One way to adjust the weights is by the Delta rule, which incorporates the learning coefficients, *lcoef,* and another parameter called momentum (M) as follows:

(N14)

$$DW_{ji}^{[s]} = lcoef * e_j^{[s]} * x_i^{[s-1]} + M * DW_{ji}.$$

The momentum (M) is a parameter which is used to smooth learning. Its function as a low-pass filter is to set an appropriate learning rate and convergence speed for the network. For more detail about artificial neural network methods, see NeuralWare, Inc. (1991).

[6] Findings using the M2 money supply resulted in the same conclusions, with negative signs for all of the Gini coefficients.

[7] In this study, variable weight (V) with relative scope connection (r) are used. The Delta weight is the result of the last change in the connection weight.

[8] This elasticity concept of the neural network is not linear, in the sense that if *GDP* changes by X percent as a result of a one percent change in the money supply, it would not imply that *GDP* changes would be 5 times X as a result of a five percent change in the money supply.

[9] It can be argued whether income concentration is a cause of changes in the *GDP*, or whether the *GDP* effects the *GC*. The Granger casualty tests for the first-difference model in the form of unrestricted equation (U) of

(U)

$$\Delta Y_t = a + \sum_{i=1}^{k} \alpha_i \Delta Y_{t-i} + \sum_{i=1}^{k} \beta_i \Delta GC_{t-i} + u_t$$

versus the restricted equation (R) of

(R)

$$\Delta Y_t = a + \sum_{i=1}^{k} \alpha_i \Delta Y_{t-i} + v_t$$

was tested. Here k is the order of the lag for Y and GC.

Similarly, the Granger casualty tests for the growth model in the form of unrestricted equation (U) of

$$\Gamma Y_t = a + \sum_{i=1}^{k} \alpha_i \Gamma Y_{t-i} + \sum_{i=1}^{k} \beta_i \Gamma GC_{t-i} + u_t \qquad (U)$$

versus the restricted equation (R) of

$$\Gamma Y_t = a + \sum_{i=1}^{k} \alpha_i \Gamma Y_{t-i} + v_t \qquad (R)$$

was tested.

According to the F-test results the null hypothesis of $a = 0$ for both the first difference and the growth version of the model with different lag order of k = 1, 2, 3, and 4 cannot be rejected.

Thus, the findings of the causality tests are not conclusive, and suggest that there is neither strong causality from the *GC* to the *GDP*, nor from the *GDP* to the *GC* for both models at one percent or even 5 percent level of significance. These results are probably due to the fact that changes in *GC* is neither the primary nor the only cause of the changes in the *GDP*. Furthermore, the results of the weak causality of *GC to GDP* could be due to the opposing effect of income distribution on the *GDP* in the short run and long run, as the findings of this research consistently imply.

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