An Investment Portfolio For OPEC: The Lagrangian Approach

MOHAMMAD SHAAF*

Introduction

The risk of fluctuation of one currency against others has increased since 1971, when the relationship between the dollar and gold was severed. Since then, the relative value of one currency against another, such as the U.S. dollar against the German mark, has been determined in the world market. That is, the forces of demand and supply primarily determine the price of a currency in terms of another.

The impact of the fluctuation of exchange rates in the world market is to create uncertainty about the future price of one currency against others and about the prices of the goods and services denominated in that currency. This problem is significant for price contracts of goods and services to be delivered in the future; for international bonds, such as Eurobonds; and for stocks.

To remedy this problem, many private agencies, corporations, and government agencies are using a composite (basket) of currencies, rather than a single currency, in their international transactions. Using a composite of currencies diminishes the risk of the fluctuations of a single currency. The degree of reduction of the risk depends on the number of the currencies in the basket, their price correlation, and the weights of each currency in the composite.

The price of OPEC oil, which is denominated in U.S. dollars, is subject to fluctuation in terms of other currencies. Given the price of OPEC oil, during a time period in which the dollar is relatively strong against other currencies, the revenue of OPEC members in terms of other currencies increases. During a period in which the dollar is relatively weak, the revenue of OPEC members in terms of other currencies decreases. Furthermore, due to these changes, the value of oil in terms of other currencies fluctuates. Therefore, OPEC member revenue is subject to a risk-of-value change in terms of other currencies.

In the early 1970's, after the official connection between the dollar and gold had been broken, OPEC negotiated with the multinational oil companies to adjust the price of OPEC oil according to relative changes of the U.S. dollar against different baskets of currencies [OPEC, 1980 (a) and (b)].

However, this switch did not go far enough to achieve the maximum stability of revenue. To accomplish that, it was necessary to give higher weights to more stable currencies and lower weights to less stable currencies. But the weights of currencies in those baskets were equal. Although stability of revenue was important for OPEC members, the emphasis was mostly on store of value rather than revenue stability.

Also, since the early 1970's, when the number of OPEC's petrodollars increased, some OPEC members had been looking for investments for their excess dollars. However, during most of the 1970's, the U.S. dollar against most other major currencies was weak. [OPEC, 1980 (a) and (b)]. Consequently, "surplus" members of OPEC, those whose economies could not absorb and consume the excess revenue (such as Saudi Arabia and Kuwait), preferred to keep their surplus funds in currencies other than the dollar.

Since the beginning of the 1980's, the opposite condition has occurred and seems to be continuing. That is, the U.S. dollar is relatively strong against other major currencies and there has been a preference for investments denominated in dollars on the part of OPEC investors.

The purpose of this article is to find the weight of each currency in a portfolio of

*Central State University

eight currencies held for investment purposes by OPEC members. It is assumed that some members of OPEC desire to invest their excess dollars in multiple currencies. These currencies are the Belgian franc, the French franc, the German mark, the Italian lira, the Japanese yen, the Swiss franc, the British pound, and the Canadian dollar. These are the currencies of the major industrial countries of the West with which OPEC trades.

Using the Lagrangian multiplier technique, the weights are found that minimizes the risk of the portfolio, measured by the variance of the value of the basket. That combination of the weights in the portfolio which minimizes its variance would be the optimum combination.[1] Before the results are explained, the methodology and the design of the model will be reviewed.

Methodology and Mathematical Model

It is assumed that an OPEC member, for example, Saudi Arabia or Kuwait, wants to invest its excess dollars in the form of a portfolio of other currencies, so that the value of the investment has a relatively stable rate of return. The problem is one of optimization—minimization of the variance of the value of the portfolio—subject to some constraints. The goal is to determine the proportional weights of each currency in a portfolio for investment purposes.

The Markowitz approach was used for the measurement of the weights of each currency [Markowitz, 1952]. Accordingly, the variance of the value of the portfolio as a whole depends on the weights as well as the covariance of the expected value of the currencies in the basket portfolio:

$$V(Portf) = \sum_{i=1}^{n} \sum_{j=1}^{n} \mathrm{W}i \cdot WJ \cdot CVij, \quad (1)$$

Where W is the weights of each currency and $CVij$ is the covariance of the expected value between two currencies i $(i = 1 \ldots 8)$ and j $(j = 1 \ldots 8)$.[2]

[1]For a discussion of an optimum currency basket for OPEC, see [Shaaf, 1984(a) and 1984(b)].

The objective is to minimize the above objective function as a measure of the risk of the portfolio, subject to the following two constraints:

(1) That the expected value of the rate of return of the portfolio over time period t increases by d. That is,

$$E(Portf) = \sum_{i=1}^{n} E(Ei) \cdot Wi = d, \quad (2)$$

where $E(Portf)$ is the expected rate of return on the portfolio and $E(Ei)$ is the expected rate of return of each currency during the time period.[3]

To keep the value of the portfolio constant over time, it is required that equation (2) be equal to zero instead of d. If OPEC members desire to increase the rate of return in the portfolio, that value can be incorporated in (2). For example, if an OPEC member wants to increase the value of its portfolio by 4 percent ($d = .04$), then equation (2) will be

$$E(Portf) = \sum_{i=1}^{n} E(Ei) \cdot Wi = .04. \quad (2)$$

The second constraint is that the sum of the weights of the currencies in the portfolio must be equal to one. That is, for the eight currency portfolio

$$\sum_{i=1}^{n} Wi = W1 + W2 + W3 + W4 + \ldots + W8 = 1. \quad (3)$$

[2]The covariance between two variables measures the magnitude of the association of the two variables. For the two currencies, Ei and Ej $(i \neq j)$, the covariance of the rate of return on these currencies is measured as:

$$CVi,j = \sum_{i=1}^{n} \sum_{j=1}^{n} (DEi - \overline{DEi})\ (DEj - \overline{DEj})\ /\ (n-1),$$

where n is the number of observation, in this case 14. DEi and DEj are the annual rate of changes of ith and jth currencies in terms of the U.S. dollar, respectively. If $i = j$, the measure is the variance. $\overline{DEi}$ and $\overline{DEj}$ are their corresponding mean.

[3]The rate of change of the value of each of eight currencies in terms of the dollar is measured as $DEi = ((Ei1 - Ei0)\ /\ Ei0) \times 100$. Where $Ei1$ and $Ei0$ are the exchange rate of the currency for two different consecutive time periods (1970 to 1983). Therefore, the expected rate of return, or mean, of each currency can be calculated as:

$$E(Ei) = \sum_{i=1}^{n} DEi/14,$$

where 14 is the number of years (i = 1970 to 1983).

Thus, the model minimizes the objective function:

$$V(Portf) = \sum_{i=1}^{n} \sum_{j=1}^{n} Wi \cdot Wj\ CVij,$$

subject to $\sum_{i=1}^{n} E(Ei)\ Wi = d,$

and

$$\sum_{i=1}^{n} W = W1 + W2 + W3 + \ldots W8 = 1.$$

Expressing equations (1), (2), and (3) as a Lagrangian multiplier equation, one obtains:

$$V(Portf) = \sum_{i=1}^{n} \sum_{j=1}^{n} Wi \cdot Wj\ CVij + L1 \left(\sum_{i=1}^{n} E(Ei)\ Wi - d\right) + L2 \sum_{i=1}^{n} Wi - 1). \quad (4)$$

To minimize (4), the partial derivative equations (first order condition) of $V(Portf)$ with respect to Wi ($i = 1, 2, \ldots 8$), $L1$, $L2$, must be equal to zero.

Giving different values for the rate of return on the basket, d, the system of equations are solved for weights, Wi ($i = 1, 2, 3, \ldots 8$) and the two Lagrangian coefficients $L1$ and $L2$. Since the restriction on the weights of the basket portfolio is that the sum of the weights be equal to 1, $\sum_{i=1}^{n} Wi = 1$, some of the weight results may be negative or greater than 1.[4] The interpretation of the negative weights, in the portfolio context, is to consider them as short-sale. This means that OPEC investors should contract the future sale of these currencies in the future markets.[5]

[4]To achieve only positive weights for other purposes for which the negative sign is not applicable, such as the price of OPEC oil, one needs to add another constraint of all weights, Wi, to be positive or zero in the model. The technique for solving this type of model is quadratic programming, rather than the Lagrangian multiplier method which has been used in this paper. The quadratic programming approach has been used for the measurement of an optimum basket of currency for OPEC. See Shaaf, "A Stable Currency Basket for OPEC: Quadratic Approach" (forthcoming). See also [Aubey and Cramer, 1977; Johnson and Zuber, 1979].

[5]To assure that the objective function (the variance of the portfolio as the measure of risk of the portfolio) is the minimum, rather than the maximum, one should test the second order condition requirement. This condition requires that all the bordered principal minor of Hessian to be positive.

Empirical Results

The annual exchange rate data from 1970 to 1983; the expected rate of return of each currency, $E(Ei)$, ($i = 1 \ldots 8$); and the variances of each of the eight currencies in terms of the U.S. dollar, are measured and shown in Table 1. The variance, $CVi, j(i = j)$, and the covariance, $CVi, j(i \neq j)$ between the ith and jth currencies are also measured and presented in Table 2.

Substituting the means, variances, and covariances in the partial derivative equations, the weights, Wi, of each currency in the portfolio were calculated and their results are shown in Table 3.

The rates of return on the portfolio were selected to be $d = 0.00, 0.01, 0.02, \ldots 0.10, 0.20, 0.30$. Accordingly, and given the magnitude of d (the rate of return on the portfolio), the weights of each currency are measured and shown in Table 3. As the table shows, some of the weights are negative. However, the sum of these weights is still equal to one. The negative weights, with their magnitudes, reflect the degree of undesirability of some of the individual currencies in the portfolio.[6]

Technically, those currencies with negative weight must be sold short by OPEC investors; i. e., these currencies are not desirable for the investors and, therefore, proportional to their weights, their sale contracts in the future markets should be in the portfolio.

Based on the results, the Canadian dollar, with the highest weight range of +79.3 to +80.1 percent, appears to be the most desirable currency. The Japanese yen, with the weight range of +20.0 to 20.1 percent, seems to be the second most desirable currency in the portfolio. Notice that the variance of the rate of change of the Canadian dollar of 13.85 is the lowest. The third most desirable currency is the Swiss franc.

The most undesirable currency in the portfolio is the Belgian franc, with the negative weight range of -13.1 to -11.0 percent. The variance of the rate of change for the latter

[6]See footnote 5.

TABLE 1

Mean and Variance of the Annual Percentage Changes of the Exchange Rates for Eight Currencies (1970-1983)

No.	Currency	Mean	Variance
1	Belgian franc	1.10	156.93
2	French franc	3.83	168.10
3	German mark	-2.33	109.19
4	Italian lira	7.42	147.42
5	Japanese yen	-2.39	79.85
6	Swiss franc	-4.42	107.01
7	British pound	6.95	1077.41
8	Canadian dollar	1.01	13.85

Source: *International Financial Statistics*, International Monetary Fund, various issues.

TABLE 2

Variance-Covariance Matrix of the Changes of the Rate of Return Between Exchange Rates (1970-9183)[a]

	1	2	3	4	5	6	7	8
1	156.93							
2	153.54	168.10						
3	123.65	121.22	109.19					
4	129.35	138.23	109.71	147.42				
5	58.93	51.57	39.84	42.60	79.85			
6	110.23	114.68	94.79	89.44	48.12	107.01		
7	20.41	-57.18	29.02	-43.85	53.09	-21.56	1077.41	
8	-5.05	-6.6	-3.68	-3.65	-10.00	-10.26	37.00	13.85

[a]1 = Belgian franc 2 = French franc 3 = German mark 4 = Italian lira
5 = Japanese yen 6 = Swiss franc 7 = British pound 8 = Canadian dollar

TABLE 3

Weights of Eight Currencies in an Investment Portfolio for OPEC With Different Rates of Return[a]

Rate of Return	Belgian franc	French franc	German mark	Italian lira	Japan yen	Swiss franc	British pound	Canadian dollar
$d = 0.00$ to $d = 0.30$	-13.1 to -11.0	-0.3 to -0.5	+3.5 to +0.6	+6.5 to +8.7	+20.0 to +20.0	+7.3 to +7.1	-3.5 to -3.6	+79.3 to +80.1

[a]The sum of each row may not be equal to 100 because of rounding.

currency is 156.93. The British pound and the French franc are the second and the third most undesirable currencies in the portfolio. The currencies with the negative weights should be sold short by OPEC investors.

The signs and the magnitude of these weights primarily depend on the variances and covariances between these currencies. Two other factors also influence the sign and the magnitude of the weights: the expected rate of return of each currency over time, $E(Ei)$, and the rate of return on the portfolio, d. Given these two latter factors, the lower (higher) the variance and covariance between these currencies, the higher (lower) their estimated weights.

Summary and Conclusions

Since the establishment of the floating exchange rate system in the early 1970's and subsequently large variations in foreign exchange rates, international financial and business institutions have been exposed to exchange rate risks. This problem is significant for price contracts of goods and services to be delivered in the future and for international investments. To cure this problem, many private agencies, corporations, and government agencies are using a composite of currencies, rather than a single currency, in their international transactions and investment.

A portfolio of eight major currencies has been selected for OPEC investment and presented in the form of an optimization function with two constraint functions. Using the Lagrangian multiplier technique, the weights of each of the eight currencies were measured.

The results indicate that the Canadian dollar is the most favorable currency and the Belgian franc is the most undesirable currency in the portfolio. The results of this study are based on *ex-post* rather than *ex-ante* data. That is, the investors should keep in mind that these results were based on 1970-83 data and that they may not necessarily reflect the future. Therefore, the adoption and the performance of a portfolio with these weights for their currencies depend on the performance of these currencies in the future, which will not necessarily follow the past path.[7]

The optimality of the portfolio in terms of minimizing the risk, as measured by the var-

[7]This is not unique in social science. The parameters of econometric models are measured and based on past data. That is, they assumed implicitly that the future follows the past path. Extremist "technicians" only use past data and do not include any other factor for their forecast of the future.

iance of the currency portfolio, depends on how much the future will be the mirror of the past in terms of the variance, covariance, and mean of the rates of change of the value of the currencies in the portfolio. Thus, investors should adjust their portfolio to the extent that future exchange rate paths deviate from the past.

REFERENCES

Robert T. Aubey and Robert H. Cramer, "The Use of International Currency Cocktails in the Reduction of Exchange Rate Risk," *Journal of Economics and Business*, 29, Winter 1977, pp. 128-35.

J. L. Evans and S. H. Archer, "Diversification and Reduction of Dispersion," *Journal of Finance*, December 1968, pp. 761-69.

International Monetary Fund, Bureau of Statistics, *Director of Trade Yearbook*, 1977-1983.

———, *International Financial Statistics, 1970 to 1983.*

R. Stafford Johnson and Richard A. Zuber, "A Model For Constructing Currency Cocktails," *Business Economics*, May 1979, pp. 9-14.

H. Markowitz, "Portfolio Analysis," *Journal of Finance*, 8, March 1952, pp. 77-91.

OPEC, *OPEC Information Booklet*, Vienna, Austria, March 1980.

———, *OPEC Chronology 1960-1980*, Vienna, Austria, June 1980.

G.V. Reklaitis, A. Ravindran, and K. M. Ragsdell, *Engineering Optimization, Methods and Applications*, John Wiley & Sons, 1983.

Mohammad Shaaf, "Strong Dollar, Low Inflation and OPEC's Terms of Trade," *Journal of Energy and Development*, Autumn 1984, pp. 121-28.

——— and James Jonish, "International Pricing of OPEC Oil: Is the Dollar Preferable to Currency Baskets?" *Central State Business Review,* Spring 1984, pp. 30-6.